

**AP Calculus AB**  
**Summer Work Packet**  
**2022-2023**

**Please Read Carefully!**

- 1) You are expected to complete this packet over the summer.
- 2) You will be held responsible for the knowledge in this packet!
- 3) There will be a test early in the year that will assess you on much of this content!
- 4) Before answering the questions on the topic, be sure to read the examples and notes to understand it fully.
- 5) If you are stuck, there are many great online resources, but Partick JMT has many YouTube videos and so does KhanAcademy.

Name: \_\_\_\_\_

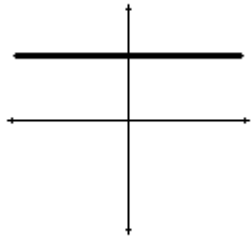


# Toolkit of Functions

Students should know the basic shape of these functions and be able to graph their transformations without the assistance of a calculator.

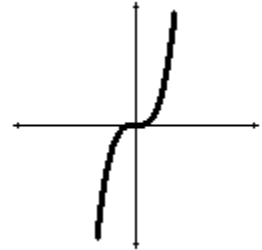
**Constant**

$$f(x) = a$$



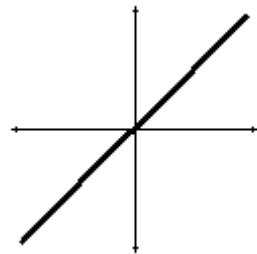
**Cubic**

$$f(x) = x^3$$



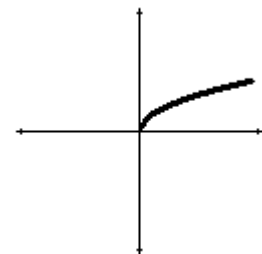
**Identity**

$$f(x) = x$$



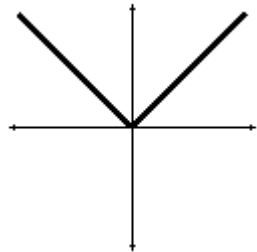
**Square Root**

$$f(x) = \sqrt{x}$$



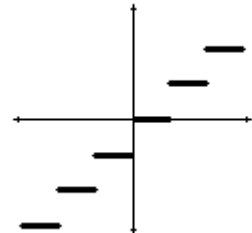
**Absolute Value**

$$f(x) = |x|$$



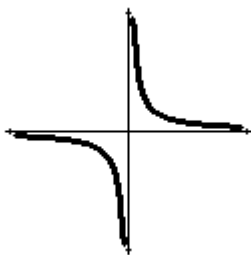
**Greatest Integer**

$$f(x) = [x]$$



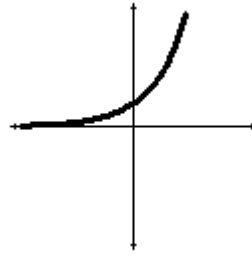
**Reciprocal**

$$f(x) = \frac{1}{x}$$



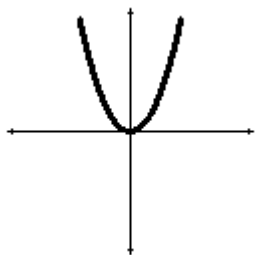
**Exponential**

$$f(x) = a^x$$



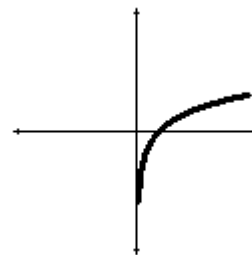
**Quadratic**

$$f(x) = x^2$$



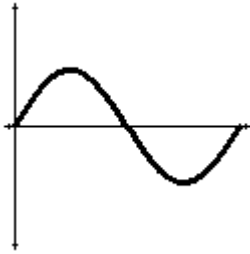
**Logarithmic**

$$f(x) = \ln x$$

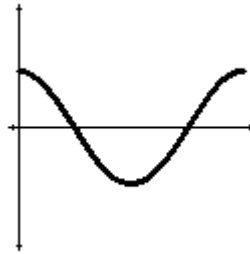


## Trig Functions

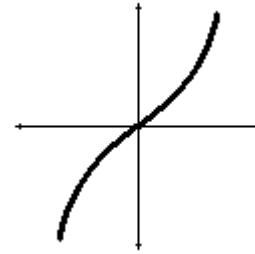
$$f(x) = \sin x$$



$$f(x) = \cos x$$



$$f(x) = \tan x$$



## Polynomial Functions:

A function  $P$  is called a polynomial if  $P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$   
Where  $n$  is a nonnegative integer and the numbers  $a_0, a_1, a_2, \dots, a_n$  are constants.

Even degree

Odd degree

Leading coefficient sign

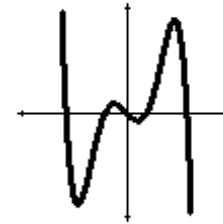
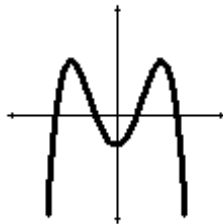
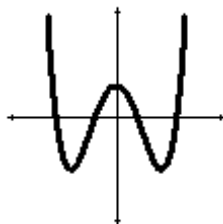
Leading coefficient sign

Positive

Negative

Positive

Negative



- Number of roots equals the degree of the polynomial.
- Number of  $x$  intercepts is less than or equal to the degree.
- Number of "turns" is less than or equal to (degree - 1).

## FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for  $x$ .

**Recall:**  $(f \circ g)(x) = f(g(x))$  OR  $f[g(x)]$  read "f of g of x" Means to plug the inside function (in this case  $g(x)$ ) in for  $x$  in the outside function (in this case,  $f(x)$ ).

**Example:** Given  $f(x) = 2x^2 + 1$  and  $g(x) = x - 4$  find  $f(g(x))$ .

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let  $f(x) = 2x + 1$  and  $g(x) = 2x^2 - 1$ . Find each.

1.  $f(2) =$  \_\_\_\_\_      2.  $g(-3) =$  \_\_\_\_\_      3.  $f(t+1) =$  \_\_\_\_\_

4.  $f[g(-2)] =$  \_\_\_\_\_      5.  $g[f(m+2)] =$  \_\_\_\_\_      6.  $[f(x)]^2 - 2g(x) =$  \_\_\_\_\_

Let  $f(x) = \sin(2x)$  Find each exactly.

7.  $f\left(\frac{\pi}{4}\right) =$  \_\_\_\_\_      8.  $f\left(\frac{2\pi}{3}\right) =$  \_\_\_\_\_

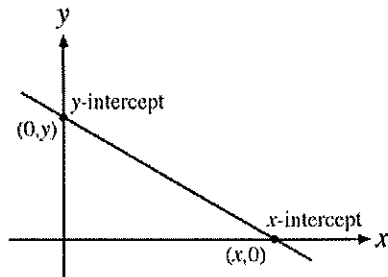
Let  $f(x) = x^2$ ,  $g(x) = 2x + 5$ , and  $h(x) = x^2 - 1$ . Find each.

9.  $h[f(-2)] =$  \_\_\_\_\_      10.  $f[g(x-1)] =$  \_\_\_\_\_      11.  $g[h(x^3)] =$  \_\_\_\_\_

## INTERCEPTS OF A GRAPH

To find the x-intercepts, let  $y = 0$  in your equation and solve.

To find the y-intercepts, let  $x = 0$  in your equation and solve.



**Example:** Given the function  $y = x^2 - 2x - 3$ , find all intercepts.

x-int. (Let  $y = 0$ )

$$0 = x^2 - 2x - 3$$

$$0 = (x - 3)(x + 1)$$

$$x = -1 \text{ or } x = 3$$

x-intercepts  $(-1, 0)$  and  $(3, 0)$

y-int. (Let  $x = 0$ )

$$y = 0^2 - 2(0) - 3$$

$$y = -3$$

y-intercept  $(0, -3)$

**Find the x and y intercepts for each.**

12.  $y = 2x - 5$

13.  $y = x^2 + x - 2$

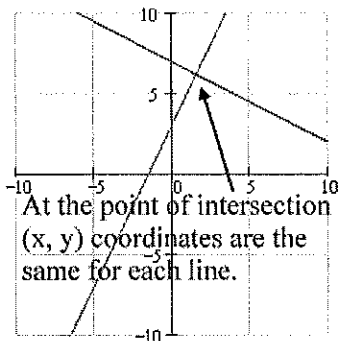
14.  $y = x\sqrt{16 - x^2}$

15.  $y^2 = x^3 - 4x$

## POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.

**Remember:** You are finding a **POINT OF INTERSECTION** so your answer is an ordered pair.



### CALCULATOR TIP

Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2<sup>nd</sup> Trace) and hit INTERSECT.

**Example:** Find all points of intersection of  $x^2 - y = 3$   
 $x - y = 1$

#### ELIMINATION METHOD

Subtract to eliminate  $y$

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ or } x = -1$$

Plug in  $x = 2$  and  $x = -1$  to find  $y$

Points of Intersection:  $(2, 1)$  and  $(-1, -2)$

#### SUBSTITUTION METHOD

Solve one equation for one variable.

$$y = x^2 - 3$$

$$y = x - 1$$

Therefore by substitution  $x^2 - 3 = x - 1$

$$x^2 - x - 2 = 0$$

From here it is the same as the other example

**Find the point(s) of intersection of the graphs for the given equations.**

16.  $x + y = 8$   
 $4x - y = 7$

17.  $x^2 + y = 6$   
 $x + y = 4$

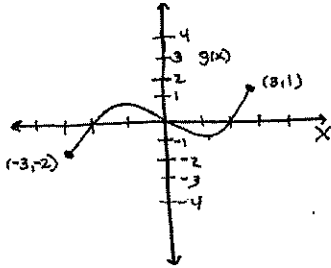
18.  $x = 3 - y^2$   
 $y = x - 1$

## DOMAIN AND RANGE

Domain – All  $x$  values for which a function is defined (input values)

Range – Possible  $y$  or Output values

### EXAMPLE 1



a) Find Domain & Range of  $g(x)$ .

The domain is the set of inputs (set of the function).  
Input values run along the horizontal axis.

The furthest left input value associated with a pt. on the graph is  $-3$ . The furthest right input values associated with a pt. on the graph is  $3$ .

So Domain is  $[-3, 3]$ , that is all reals from  $-3$  to  $3$ .

The range represents the set of output values for the function. Output values run along the vertical axis.

The lowest output value of the function is  $-2$ . The highest is  $1$ . So the range is  $[-2, 1]$ , all reals from  $-2$  to  $1$ .

### EXAMPLE 2

Find the domain and range of  $f(x) = \sqrt{4-x^2}$   
Write answers in interval notation.

#### DOMAIN

For  $f(x)$  to be defined  $4-x^2 \geq 0$ .

This is true when  $-2 \leq x \leq 2$

Domain:  $[-2, 2]$

#### RANGE

The solution to a square root must always be positive thus  $f(x)$  must be greater than or equal to  $0$ .

Range:  $[0, \infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

19.  $f(x) = x^2 - 5$

20.  $f(x) = -\sqrt{x+3}$

21.  $f(x) = 3 \sin x$

22.  $f(x) = \frac{2}{x-1}$

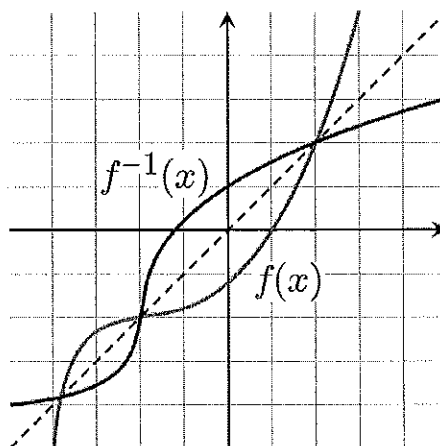


## INVERSES

To find the inverse of a function, simply switch the  $x$  and the  $y$  and solve for the new “ $y$ ” value. Recall  $f^{-1}(x)$  is defined as the inverse of  $f(x)$

### Example 1:

$f(x) = \sqrt[3]{x+1}$	Rewrite $f(x)$ as $y$
$y = \sqrt[3]{x+1}$	Switch $x$ and $y$
$x = \sqrt[3]{y+1}$	Solve for your new $y$
$(x)^3 = (\sqrt[3]{y+1})^3$	Cube both sides
$x^3 = y+1$	Simplify
$y = x^3 - 1$	Solve for $y$
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation



Find the inverse for each function.

23.  $f(x) = 2x + 1$

24.  $f(x) = \frac{x^2}{3}$

25.  $g(x) = \frac{5}{x-2}$

26.  $y = \sqrt{4-x} + 1$

27. If the graph of  $f(x)$  has the point  $(2, 7)$  then what is one point that will be on the graph of  $f^{-1}(x)$ ?

28. Explain how the graphs of  $f(x)$  and  $f^{-1}(x)$  compare.

## EQUATION OF A LINE

**Slope intercept form:**  $y = mx + b$

**Vertical line:**  $x = c$  (slope is undefined)

**Point-slope form:**  $y - y_1 = m(x - x_1)$

**Horizontal line:**  $y = c$  (slope is 0)

\* LEARN! We will use this formula frequently!

**Example:** Write a linear equation that has a slope of  $\frac{1}{2}$  and passes through the point (2, -6)

**Slope intercept form**

**Point-slope form**

$$y = \frac{1}{2}x + b$$

Plug in  $\frac{1}{2}$  for  $m$

$$y + 6 = \frac{1}{2}(x - 2)$$

Plug in all variables

$$-6 = \frac{1}{2}(2) + b$$

Plug in the given ordered

$$y = \frac{1}{2}x - 7$$

Solve for  $y$

$$b = -7$$

Solve for  $b$

$$y = \frac{1}{2}x - 7$$

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.
30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.
31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of  $\frac{2}{3}$ .
32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line  $y = \frac{5}{6}x - 1$ .
33. Use point-slope form to find a line perpendicular to  $y = -2x + 9$  passing through the point (4, 7).
34. Find the equation of a line passing through the points (-3, 6) and (1, 2).
35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)



## TRIGONOMETRIC EQUATIONS

Solve each of the equations for  $0 \leq x < 2\pi$ .

37.  $\sin x = -\frac{1}{2}$

38.  $2 \cos x = \sqrt{3}$

39.  $4 \sin^2 x = 3$

40.  $2 \cos^2 x - 1 - \cos x = 0$  \*Factor

\*\*Recall  $\sin^2 x = (\sin x)^2$

\*\*Recall if  $x^2 = 25$  then  $x = \pm 5$

## TRANSFORMATION OF FUNCTIONS

$$h(x) = f(x) + c$$

Vertical shift  $c$  units up

$$h(x) = f(x - c)$$

Horizontal shift  $c$  units right

$$h(x) = f(x) - c$$

Vertical shift  $c$  units down

$$h(x) = f(x + c)$$

Horizontal shift  $c$  units left

$$h(x) = -f(x)$$

Reflection over the x-axis

41. Given  $f(x) = x^2$  and  $g(x) = (x-3)^2 + 1$ . How does the graph of  $g(x)$  differ from  $f(x)$ ?

42. Write an equation for the function that has the shape of  $f(x) = x^3$  but moved six units to the left and reflected over the x-axis.

43. If the ordered pair  $(2, 4)$  is on the graph of  $f(x)$ , find one ordered pair that will be on the following functions:

a)  $f(x) - 3$

b)  $f(x - 3)$

c)  $2f(x)$

d)  $f(x - 2) + 1$

e)  $-f(x)$

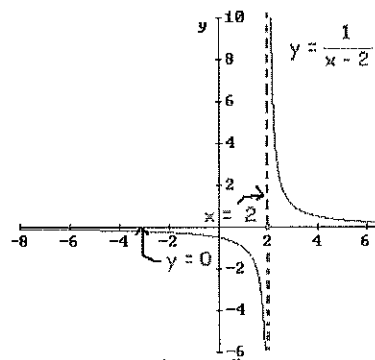
## VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).

Write a vertical asymptotes as a line in the form  $x =$

Example: Find the vertical asymptote of  $y = \frac{1}{x-2}$

Since when  $x = 2$  the function is in the form  $1/0$  then the vertical line  $x = 2$  is a vertical asymptote of the function.



44.  $f(x) = \frac{1}{x^2}$

45.  $f(x) = \frac{x^2}{x^2 - 4}$

46.  $f(x) = \frac{2+x}{x^2(1-x)}$

47.  $f(x) = \frac{4-x}{x^2 - 16}$

48.  $f(x) = \frac{x-1}{x^2 + x - 2}$

49.  $f(x) = \frac{5x+20}{x^2 - 16}$

## HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

**Case I.** Degree of the numerator is less than the degree of the denominator. The asymptote is  $y = 0$ .

Example:  $y = \frac{1}{x-1}$  (As  $x$  becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at  $y = 0$ .

**Case II.** Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Example:  $y = \frac{2x^2 + x - 1}{3x^2 + 4}$  (As  $x$  becomes very large or very negative the value of this function will approach  $2/3$ ). Thus there is a horizontal asymptote at  $y = \frac{2}{3}$ .

**Case III.** Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example:  $y = \frac{2x^2 + x - 1}{3x - 3}$  (As  $x$  becomes very large the value of the function will continue to increase and as  $x$  becomes very negative the value of the function will also become more negative).

**Determine all Horizontal Asymptotes.**

50.  $f(x) = \frac{x^2 - 2x + 1}{x^3 + x - 7}$

51.  $f(x) = \frac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

52.  $f(x) = \frac{4x^2}{3x^2 - 7}$

53.  $f(x) = \frac{(2x-5)^2}{x^2 - x}$

54.  $f(x) = \frac{-3x+1}{\sqrt{x^2+x}}$  \* Remember  $\sqrt{x^2} = \pm x$

\*This is very important in the use of limits.\*

## EXPONENTIAL FUNCTIONS

**Example: Solve for x**

$$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$$

$$(2^2)^{x+1} = (2^{-1})^{3x-2} \quad \text{Get a common base}$$

$$2^{2x+2} = 2^{-3x+2} \quad \text{Simplify}$$

$$2x+2 = -3x+2 \quad \text{Set exponents equal}$$

$$x = 0 \quad \text{Solve for x}$$

**Solve for x:**

$$55. 3^{3x+5} = 9^{2x+1}$$

$$56. \left(\frac{1}{9}\right)^x = 27^{2x+4}$$

$$57. \left(\frac{1}{6}\right)^x = 216$$

## LOGARITHMS

The statement  $y = b^x$  can be written as  $x = \log_b y$ . They mean the same thing.

**REMEMBER: A LOGARITHM IS AN EXPONENT**

Recall  $\ln x = \log_e x$

The value of  $e$  is 2.718281828... or  $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$

**Example: Evaluate the following logarithms**

$$\log_2 8 = ?$$

In exponential form this is  $2^? = 8$

Therefore  $? = 3$

Thus  $\log_2 8 = 3$

**Evaluate the following logarithms**

$$58. \log_7 7$$

$$59. \log_3 27$$

$$60. \log_2 \frac{1}{32}$$

$$61. \log_{25} 5$$

$$62. \log_9 1$$

$$63. \log_4 8$$

$$64. \ln \sqrt{e}$$

$$65. \ln \frac{1}{e}$$

## PROPERTIES OF LOGARITHMS

$$\log_b xy = \log_b x + \log_b y$$

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$

$$\log_b x^y = y \log_b x$$

$$b^{\log_b x} = x$$

Examples:

Expand  $\log_4 16x$

$$\log_4 16 + \log_4 x$$

$$2 + \log_4 x$$

Condense  $\ln y - 2 \ln R$

$$\ln y - \ln R^2$$

$$\ln \frac{y}{R^2}$$

Expand  $\log_2 7x^5$

$$\log_2 7 + \log_2 x^5$$

$$\log_2 7 + 5 \log_2 x$$

Use the properties of logarithms to evaluate the following

66.  $\log_2 2^5$

67.  $\ln e^3$

68.  $\log_2 8^3$

69.  $\log_3 \sqrt[5]{9}$

70.  $2^{\log_2 10}$

71.  $e^{\ln 8}$

72.  $9 \ln e^2$

73.  $\log_9 9^3$

74.  $\log_{10} 25 + \log_{10} 4$

75.  $\log_2 40 - \log_2 5$

76.  $\log_2 (\sqrt{2})^5$



## EVEN AND ODD FUNCTIONS

**Recall:**

*Even functions are functions that are symmetric over the y-axis.*

*To determine algebraically we find out if  $f(x) = f(-x)$*

*(\*Think about it what happens to the coordinate  $(x, f(x))$  when reflected across the y-axis\*)*

*Odd functions are functions that are symmetric about the origin.*

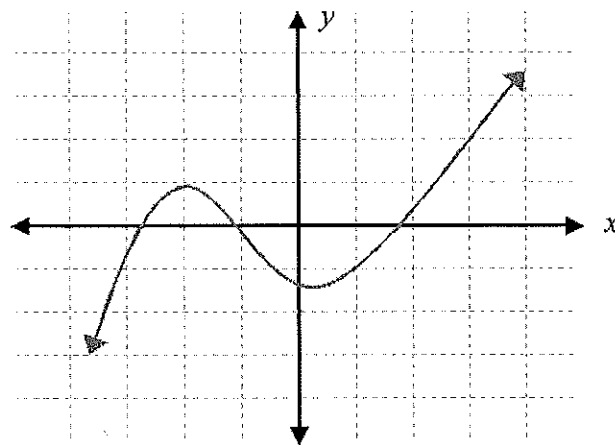
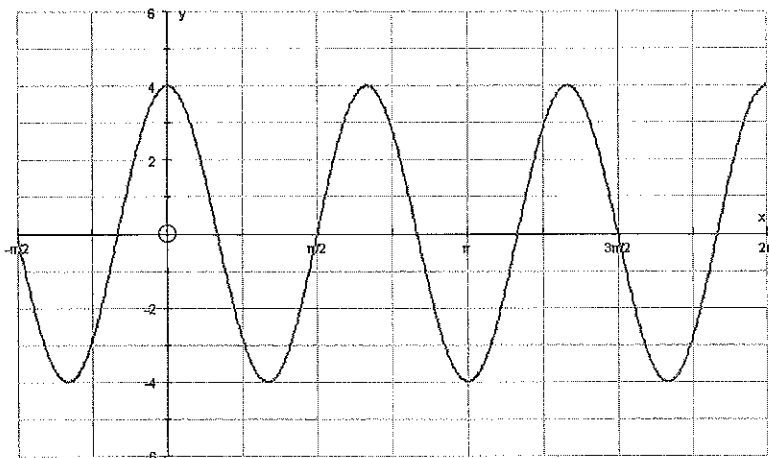
*To determine algebraically we find out if  $f(-x) = -f(x)$*

*(\*Think about it what happens to the coordinate  $(x, f(x))$  when reflected over the origin\*)*

State whether the following graphs are even, odd or neither, show ALL work.

77. \_\_\_\_\_

78. \_\_\_\_\_



79. \_\_\_\_\_

$$f(x) = 2x^4 - 5x^2$$

80. \_\_\_\_\_

$$g(x) = x^5 - 3x^3 + x$$

81. \_\_\_\_\_

$$h(x) = 2x^2 - 5x + 3$$

82. \_\_\_\_\_

$$j(x) = 2 \cos x$$

83. \_\_\_\_\_

$$k(x) = \sin x + 4$$

84. \_\_\_\_\_

$$l(x) = \cos x - 3$$

